Introduction to Focus Issue: Lagrangian Coherent Structures

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The topic of Lagrangian coherent structures (LCS) has been a rapidly growing area of research in nonlinear dynamics for almost a decade. It provides a means to rigorously define and detect transport barriers in dynamical systems with arbitrary time dependence and has a wealth of applications, particularly to fluid flow problems. Here, we give a short introduction to the topic of LCS and review the new work presented in this Focus Issue. © *2010 American Institute of Physics*. [doi:10.1063/1.3278173]

The concept and study of Lagrangian coherent structures (LCS) have evolved from a need to formally define intrinsic structures within fluid flows that govern flow transport. Roughly speaking, LCS are distinguished material lines or surfaces that delineate regions of fluid for which the long-term evolution of a tracer particle is qualitatively very different. The challenge is to develop efficient mathematical tools for identifying, and perhaps predicting, the presence and form of these structures in complex numerical and experimental data sets, which are nowadays commonplace in fluid dynamics research. This ability significantly advances our capability to both understand and exploit fluid flows in engineering and natural systems.

It has long been recognized that coherent structures are an intrinsic property of all manner of fluid flows. Leonardo da Vinci sketched the form of various flow fields over objects in a flowing stream, providing the earliest reference to the importance of vortices in fluid motion.[1] More recently, ideas concerning the decomposition of flow fields into a set of basis coherent structures, and the consequent modeling of the flow dynamics using low-dimensional models with the accompanying tools of dynamical systems theory, have provided fundamental breakthroughs in flow control and understanding the transition to turbulence.[2] Nevertheless, until the recent advent of LCS, there was still concern on how to objectively define a coherent structure as commonplace as a vortex, given that many of the existing criteria rely on Eulerian quantities that are not necessarily frame invariant.

A principle behind the development of the theory of LCS is to establish frame-independent criteria that permit unambiguous definition of coherent structures based on their influence on flow transport. To that end, when defining LCS, fluid motion is viewed from a Lagrangian rather than an Eulerian perspective. Perhaps the simplest (but not always the most reliable) tool for identifying LCS is the finite-time Lyapunov exponent (FTLE), which characterizes the rate of separation of neighboring trajectories over a finite-time interval. The FTLE is typically calculated by seeding a velocity field with tracer particles and integrating their positions forward in time. At each location, the FTLE is the maximum eigenvalue in the Jacobian of the local flow map, which maps initial to final particle positions over the finite-time interval. LCS then generally coincide with maximum ridges in the FTLE field, corresponding to structures responsible for the greatest stretching of particle paths.

The concept and terminology for LCS were introduced by Haller[3,4] and Haller and Yuan,[5] who presented mathematical criteria for the existence of finite time attracting and repelling material surfaces (i.e., finite-time hyperbolic invariant manifolds) in flows with arbitrary time dependence. It was shown that by integrating particle trajectories in both forward and backward times, diagnostic quantities (such as FTLE and hyperbolicity time) can be calculated, from which repelling and attracting LCS can be extracted. The definition of LCS was then refined to require that they be uniformly hyperbolic.[6] This led to a criterion that enables one to distinguish between FTLE ridges that are truly attracting or repelling materials lines, and FTLE ridges that simply indicate lines of high shear. It was also proven that LCS can be reliably detected even in the face of large errors in velocity field data, so long as those errors remain spatially and temporarily localized. Thereafter, Shadden *et al.*[7] showed that although ridges of the FTLE field need not be exactly advected with the flow, the net flux across a ridge is typically negligible, and LCS can indeed be approximated by evolving maximum ridges of FTLE fields with a sufficiently long integration time.

With the theory of LCS on a sound mathematical footing, there has been an explosion of related research. Significant effort has been directed toward improving computational efficiency since the necessary computational time can be prohibitively long. Notable improvements in computational time have been found using adaptive mesh refinement.[8] More robust criteria for extracting FTLE ridges have been developed and applied to experimental data from turbulent fluid flows[9] and LCS techniques have been extended to n-dimensional flows.[10] There have been a wealth of studies applying LCS to such diverse problems as pollution control strategies in the ocean,[11] unsteady flow separation,[12] blood flow,[13] jellyfish predation,[14] and inertial particle dynamics in a hurricane,[15] to name but a few examples.

This Focus Issue provides a timely opportunity to present the state of the art in LCS, which has evolved to become one of the most exciting avenues of research in dynamical systems. There is an established literature, yet the topic is still quite new and only just entering the scientific mainstream, as evidenced by recent feature articles in the New York Times[16] and The Economist.[17] The articles contributed to this Focus Issue strike a balance between providing a review of the topic and presenting the latest results in the field, the latter covering everything from new theoretical developments and improved algorithms for LCS computation to new and exciting areas of application.

On computational matters, Brunton and Rowley[18] present new and efficient methods for computing FTLE in unsteady flows. Their method approximates the underlying particle flow map, which enables elimination of redundant particle integrations when calculating time-evolving LCS. Lipinski and Mohseni[19] take a different approach and develop a ridge-tracking algorithm for following LCS, taking advantage of their spatial coherence and avoiding unnecessary computations away from the ridges. In both cases, an order of magnitude in computational savings is reported over traditional methods.

With regard to theoretical development, Lekien and Ross[20] generalize the concept of FTLE and LCS to arbitrary Riemannian manifolds, which are the more natural mathematical setting for many dynamical systems. This facilitates application of LCS concepts to transport along isopycnal surfaces in the ocean and large-scale mixing in the (curved) atmosphere. The effect of spatial and temporal resolution and random errors on LCS is investigated further by Olcay *et al.*,[21] and Ross *et al.*[22] present a method for obtaining dynamical boundaries using only trajectories reconstructed from time series, applying the ideas to problems in musculoskeletal biomechanics, thereby extending LCS ideas to more general (nonfluid) problems that concern dynamical boundaries in a phase space. This is also the case for the study of Tang and Peacock,[23] who extract LCS in the energy-flux field of oceanic internal waves, seeking a means to identify internal wave attractors.

Several contributors to this Focus Issue have identified important new flow problems where LCS provide new and fundamental insight. Tang *et al.*[24] recognize that locating LCS on a spatially limited domain, which is usually the case for geophysical data, presents a challenge because the domain boundaries inevitably appear as attractors. A finite-domain FTLE method is therefore conceived and the technique apply to the analysis of velocity field data from aircraft landing at Hong Kong International Airport. Eldredge and Chong[25] apply LCS ideas to biolocomotion problems, providing new insight into the vortex-shedding mechanisms that play an important role in unsteady aerodynamics. Green *et al.*[26] also study bioinspired flows and demonstrate that dynamical changes in the downstream flow field behind pitching panels produce corresponding qualitative changes in LCS, providing important evidence of the connection between the structure of LCS and the dynamic state of a fluid system.

For biological systems, Lukens *et al.*,[27] motivated by the desire to understand the fluid flow within the airway surface liquid of the lung, use LCS to observe ciliary transport. The computed LCS uncover a barrier that separates a recirculation region of fluid that remains near a beating cilium from fluid that is advected downstream. Shadden *et al.*[28] advance their pioneering work on using LCS to improve insight into the transport mechanics of blood flow downstream of a valve, with the goal of aiding clinical decision making. Furthermore, O'Farrell and Dabiri[29] identify that LCS provide a practical criterion for identifying vortex-ring pinch-off, being indicated by the appearance of a new disconnected LCS and the termination of the original LCS.

As research in LCS advances, there is inevitably evolution in the scope of the field. Beron-Vera *et al.*[30] use the term LCS to describe invariant tori in certain classes of two-dimensional incompressible flows. Like stable and unstable manifolds, these tori serve as transport barriers, and structures like these are often present in geophysical flows where zonal jets are present. Sapsis and Haller[31] investigate smooth deformations of invariant tori in three-dimensional steady and two-dimensional unsteady flows, both of which predict inertial particle clustering due to inertial LCS. Finally, Thiffeault[32] takes a new approach and uses tools from Braid theory and surface mapping to interpret the dynamics of particles and potentially uncovered LCS.

Exciting times lie ahead for the development of LCS and their application to real-world problems. One area of interest is to establish closer connections between the existence and form of these structures and the dynamic state of fluids, with perhaps the possibility of using LCS as building blocks for the description of fluid flows. After nearly a decade of research thus far, we look forward to the next decade with great anticipation.

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